

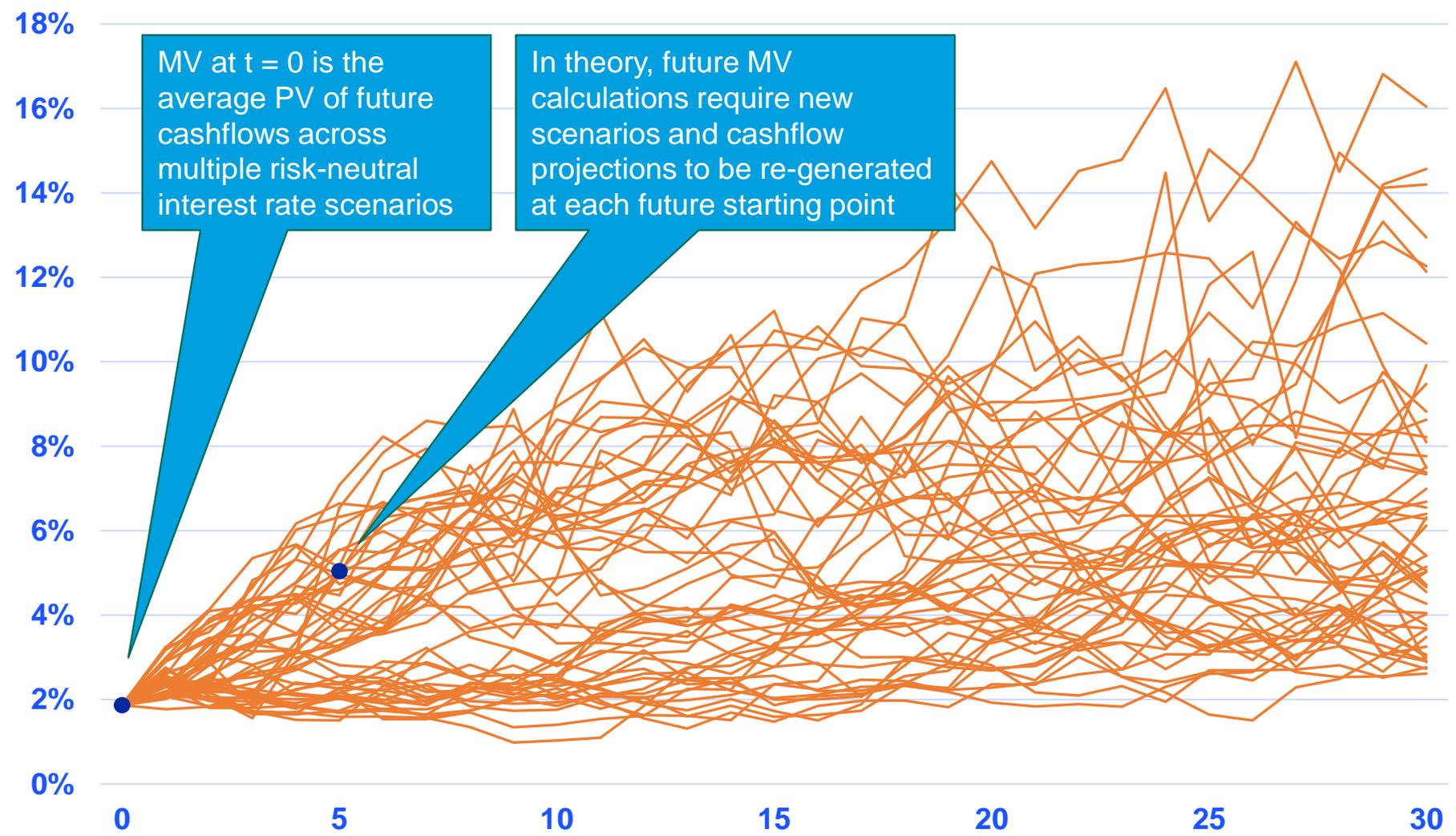
# Projecting Structured Asset MVs in AXIS

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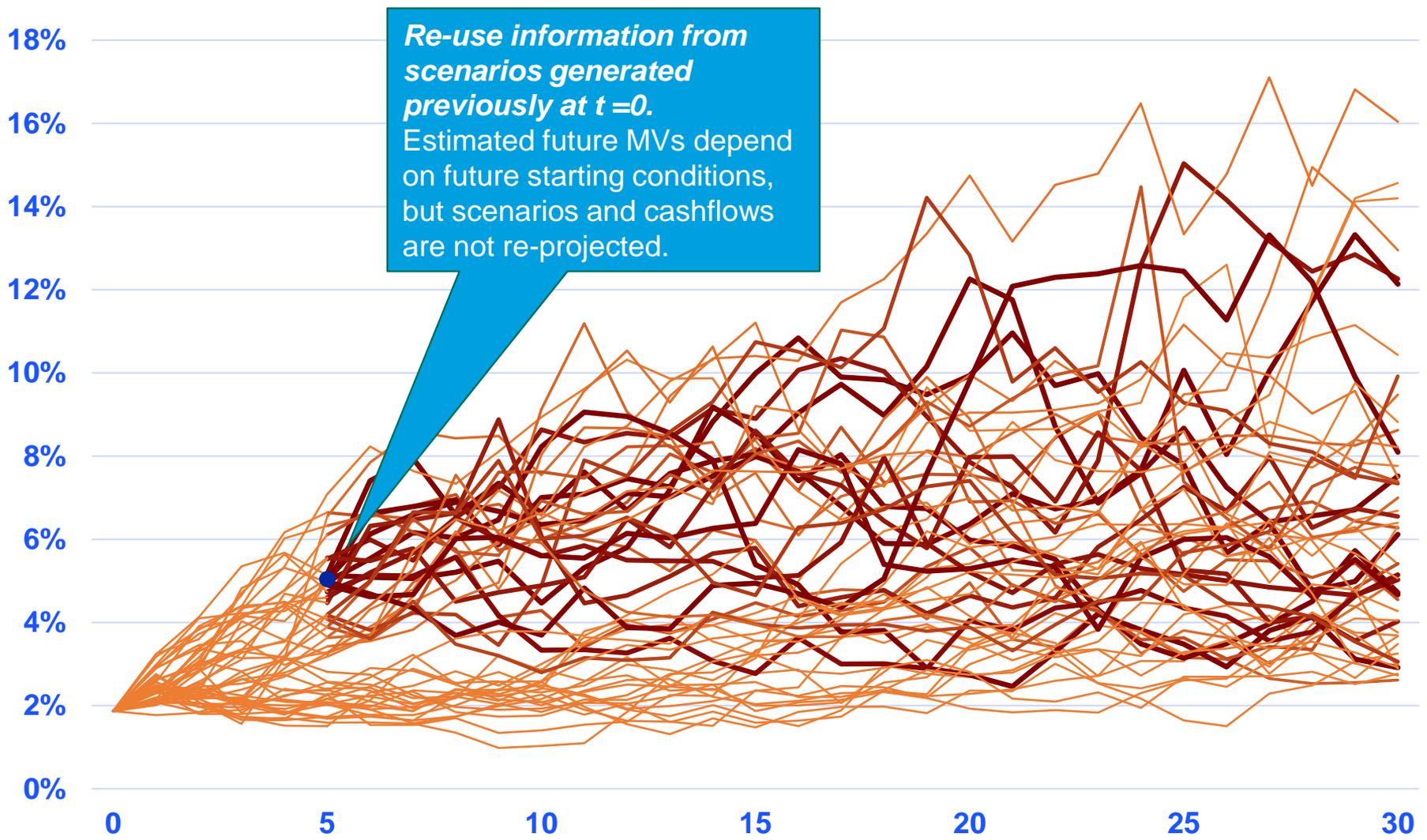
## Background

- » AXIS is a multi-year projection system for life insurers' assets and liabilities
  - Projections include future cashflows, earnings and balance sheets (BV and MV)
  - Based on user-specified economic scenarios
- » AXIS is able to link to MA's Structured Finance API to project asset cashflows, but the SF API is not designed to produce future MV projections
  - SF API can calculate MVs at the projection start date, but not at future dates.
- » In theory, accurate MV projections require nested Monte Carlo simulations at each future valuation date. In practice this requires more computational resources than most users have available.
  - Currently, AXIS calculates future MVs by using the SF API to project cashflows along a single implied-forward interest rate path.
- » MA has now developed a methodology for approximating nested Monte Carlo MV calculations in AXIS with dramatically improved run-times.
  - Also provides rapid estimates of effective duration and convexity
  - Based on proxy functions, calibrated using SF API cashflow projections
  - Implementation is currently in progress

# Illustrative Example: Simulated UST 10y Rates



# Alternative to Nested Monte Carlo Simulation



# Proxy Functions – Least Squares Monte Carlo

**Objective:** Estimate  $MV_t$  at some future time  $t > 0$  as a function of a given vector of state variables  $\mathbf{x}_t$  (future UST rates, outstanding principal, etc.)

In general,  $MV_t = \mathbb{E}^Q(PVFC_t | \mathbf{x}_t)$  where  $PVFC_t$  is the present value of future cashflows as seen at time  $t$ .

## Algorithm:

- » Estimate  $MV_0$  by generating Monte Carlo simulations of  $\mathbf{x}_{i,t}$  and  $PVFC_{i,t}$  starting at  $t = 0$ . Then  $MV_0$  is the mean of the simulated values of  $PVFC_{i,0}$ .
- » Assume that  $MV_t \approx f(\mathbf{x}_t; \boldsymbol{\theta})$  where  $f$  is a given parametric function (a proxy function) and  $\boldsymbol{\theta}$  is vector of function parameters
- » Since  $MV_t = \mathbb{E}^Q(PVFC_t | \mathbf{x}_t)$ , the parameters  $\boldsymbol{\theta}$  can be estimated by least squares regression of the simulated values of  $PVFC_{i,t}$  against  $f(\mathbf{x}_{i,t}; \boldsymbol{\theta})$ .
  - In effect, the simulated values of  $\mathbf{x}_{i,t}$  and  $PVFC_{i,t}$  provide a very large number of very poor estimates of  $\mathbb{E}^Q(PVFC_t | \mathbf{x}_t)$ .
  - Solve for  $\boldsymbol{\theta}$  to minimize  $\sum_{i,t} (PVFC_{i,t} - f(\mathbf{x}_{i,t}; \boldsymbol{\theta}))^2$
- » Well-established technique (Longstaff & Schwartz, 2001)

# Model Details

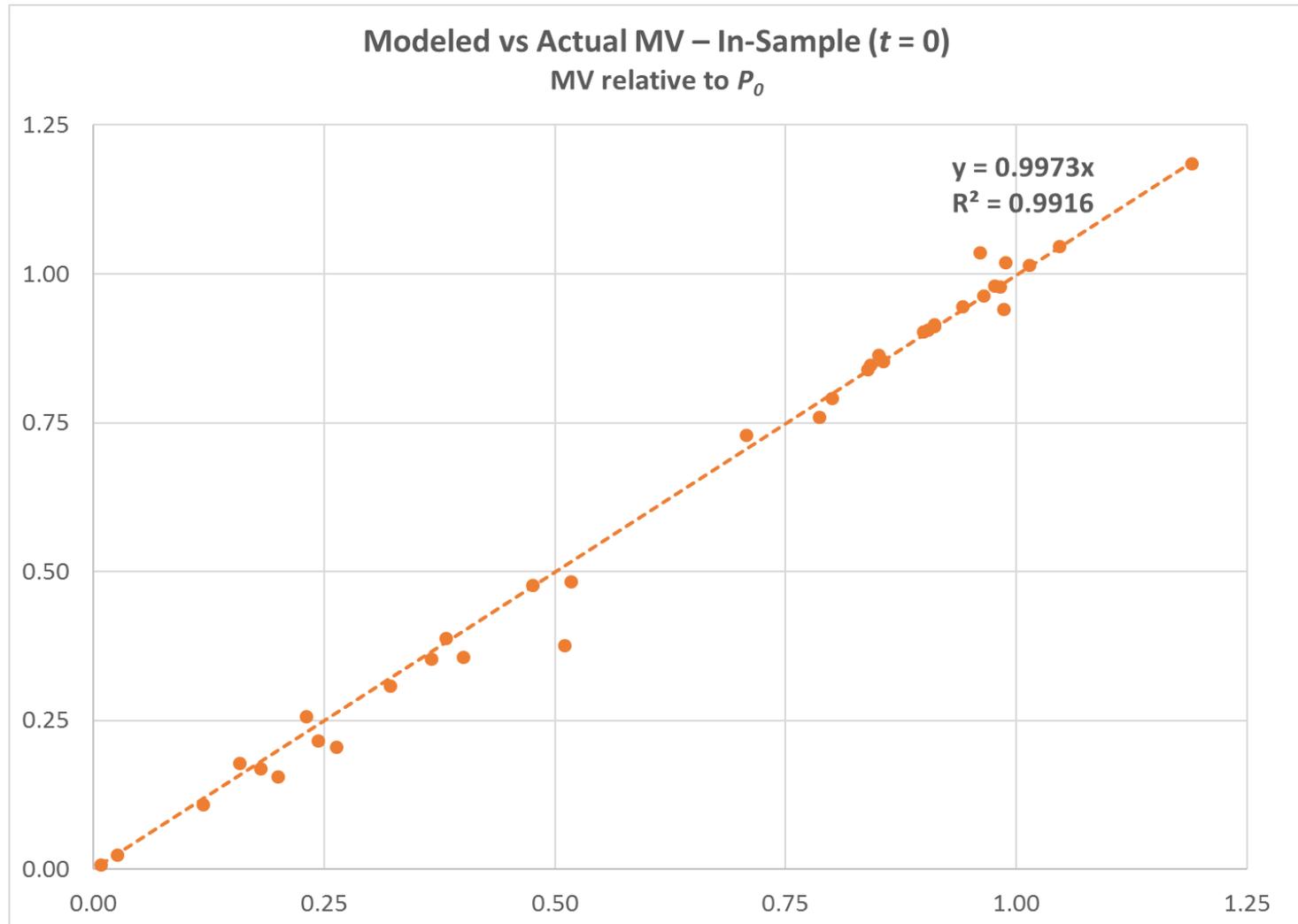
- » Assume that  $g(MV_t) \approx \boldsymbol{\theta} \cdot \mathbf{x}_t$  where  $g(MV_t) = \log(\exp(MV_t) - 1)$ 
  - Implies that  $MV_t \approx g^{-1}(\boldsymbol{\theta} \cdot \mathbf{x}_t) = \log(\exp(\boldsymbol{\theta} \cdot \mathbf{x}_t) + 1)$
  - Ensures that the modeled value of  $MV_t$  is always positive even if  $\boldsymbol{\theta} \cdot \mathbf{x}_t$  is negative, since  $g^{-1}(y) \rightarrow y$  as  $y \rightarrow +\infty$  but  $g^{-1}(y) \rightarrow 0$  as  $y \rightarrow -\infty$ .
- » This is a **generalized linear model**
  - A generalized linear model is *not* a linear model. The link function  $g$  is non-linear, and the state vector  $\mathbf{x}_t$  can include polynomial and interaction terms.
- » Predictor variables (state variables):
  - $a_t = P_t/P_0$  where  $P_t$  is the outstanding bond principal at time  $t$
  - $b_t = 10\text{y US Treasury rate}$
  - $c_t = 1\text{y}/10\text{y US Treasury spread}$
  - $d_t = 6\text{m USD LIBOR}/\text{Treasury spread}$
  - $\mathbf{x}_t = \{1, t, t^2, a_t, b_t, c_t, d_t, \dots, t^m a_t^n, t^m b_t^n, t^m c_t^n, t^m d_t^n\}$  where  $m, n \leq 2$  (27 terms)
    - ›  $PVFC_{i,t}$  is discounted using Treasury rates plus a spread, but for a given bond the discount spread is constant for all  $i$  and  $t$ , and is not an explicit predictor variable
- » Parameters  $\boldsymbol{\theta}$  can be constrained to match  $g^{-1}(\boldsymbol{\theta} \cdot \mathbf{x}_0)$  to a given value of  $MV_0$

# Model Calibration Process

- » Tested on 44 ABS and RMBS CUSIPs from actual client portfolios
  - Selected CUSIPs all had highly variable cashflows
- » Moody's Analytics ESG 2-factor BK model used to generate 1,000 risk-neutral interest rate paths for US Treasuries and USD LIBOR
- » SAV API and Moody's Analytics MPA/PA prepayment and default models used to project cashflows and principal balances along each path
  - Projected at monthly intervals until maturity or until  $P_t < 0.05 P_0$
  - Average of about 50,000 simulated values of  $x_{i,t}$  and  $PVFC_{i,t}$  for each CUSIP
- » Solved for  $\theta$  using a standard GLM package
  - Used stepwise GLM regression to identify irrelevant predictors. Not all predictors were relevant to all CUSIPs, but each predictor was relevant to the majority of CUSIPs.
  - Also considered ordinary least squares (OLS) regression as an alternative to GLM regression. OLS is easier to implement, but results were less accurate.
    - › OLS solves for  $\theta$  to minimize  $\sum_{i,t} (g(PVFC_{i,t}) - \theta \cdot x_{i,t})^2$  instead of  $\sum_{i,t} (PVFC_{i,t} - g^{-1}(\theta \cdot x_{i,t}))^2$ .
- » Regression  $R^2$  varied from 20.8% to 93.2%
  - A low  $R^2$  does not necessarily imply a poor quality model. The purpose of the model is to estimate  $MV_t = \mathbb{E}^Q(PVFC_t | x_t)$ , not  $PVFC_{i,t}$ . The latter is a random variable that can have many different values for given predictor vector  $x_t$ .

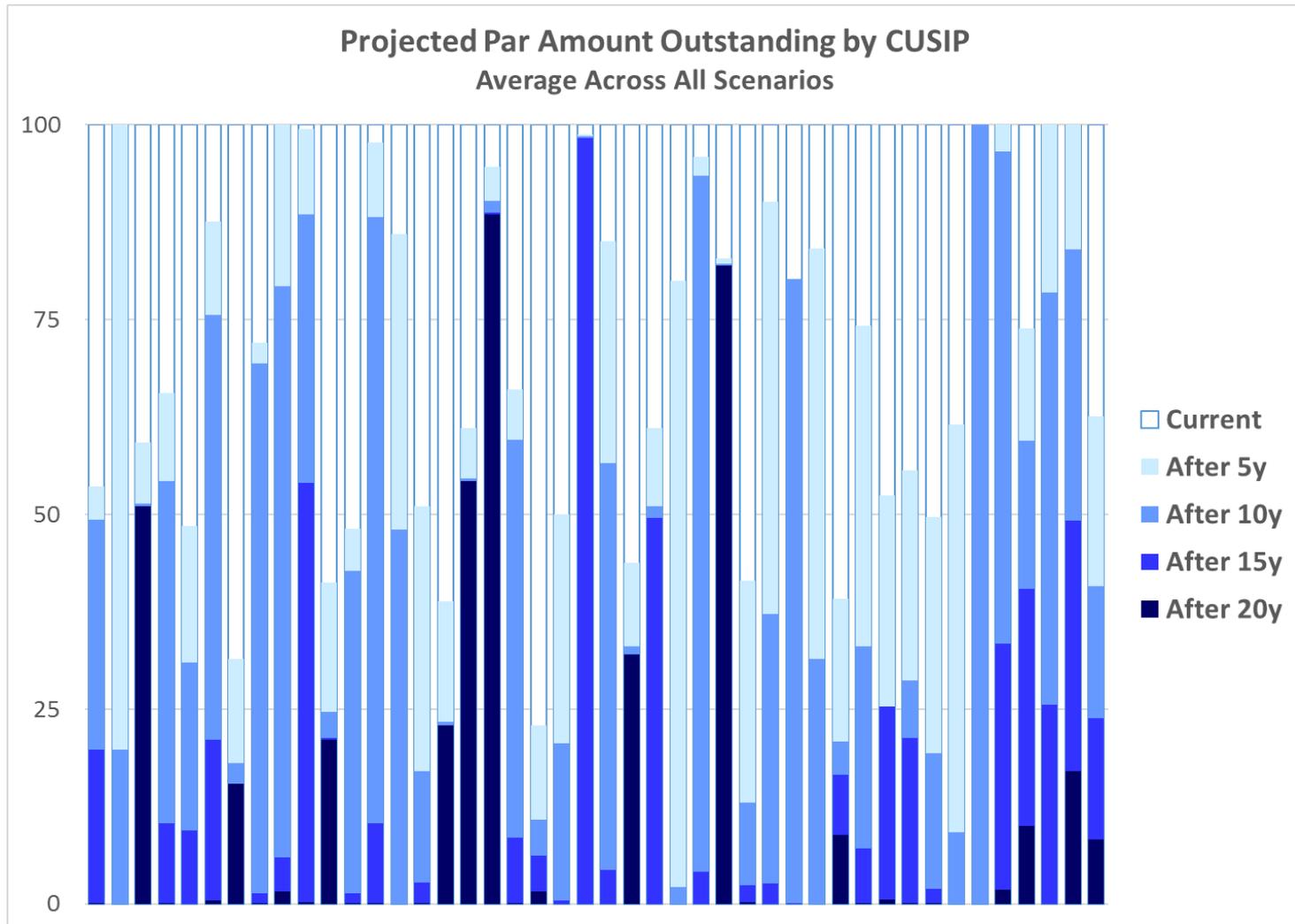
# In-Sample Testing

- » Compared modeled and actual values of  $MV_0$  (mean of  $PVFC_{i,0}$ ) for each CUSIP



# Projected Par Amounts Outstanding

» Most CUSIPs in test sample pay down over 5-15 years

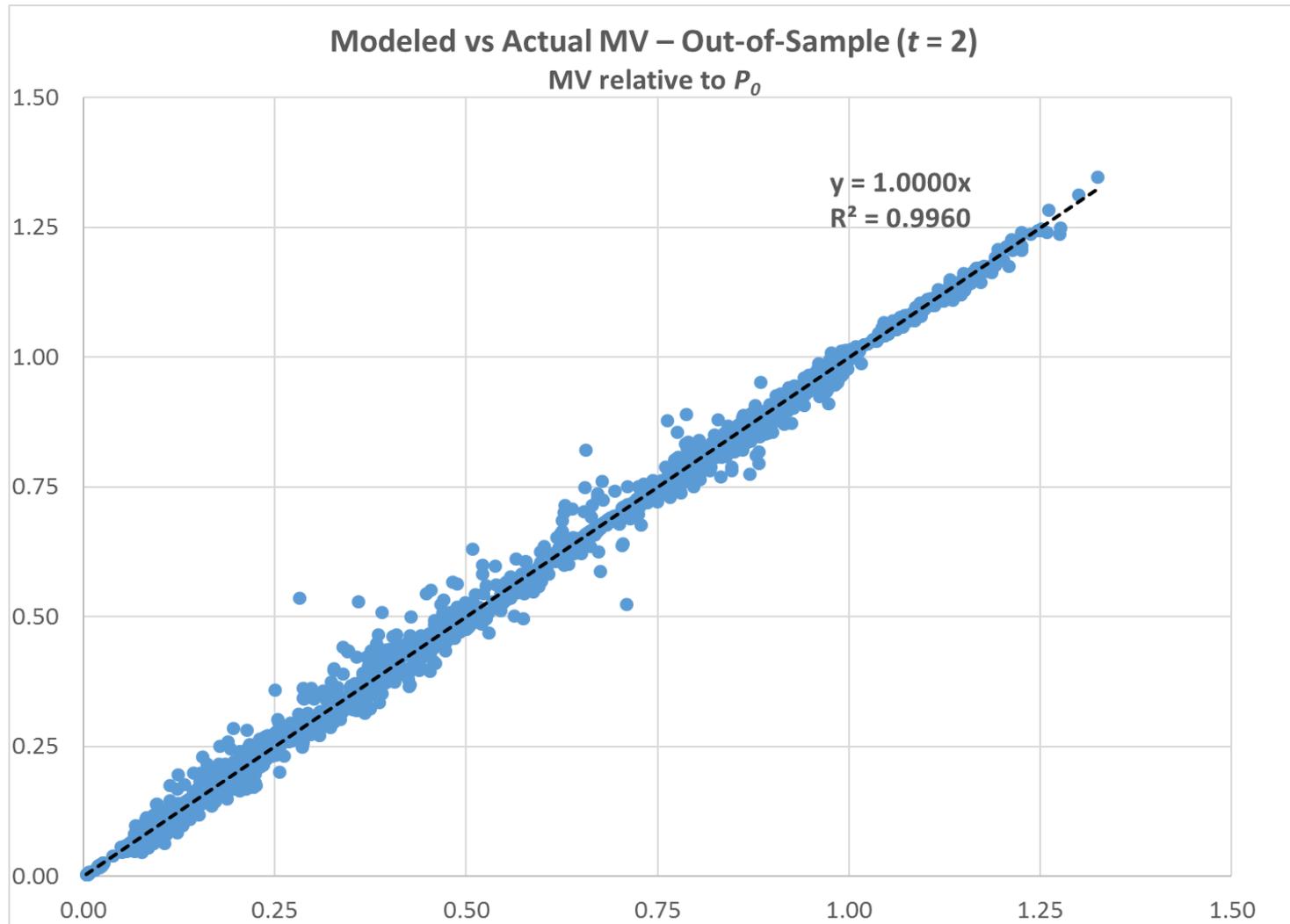


## Out-of-Sample Testing

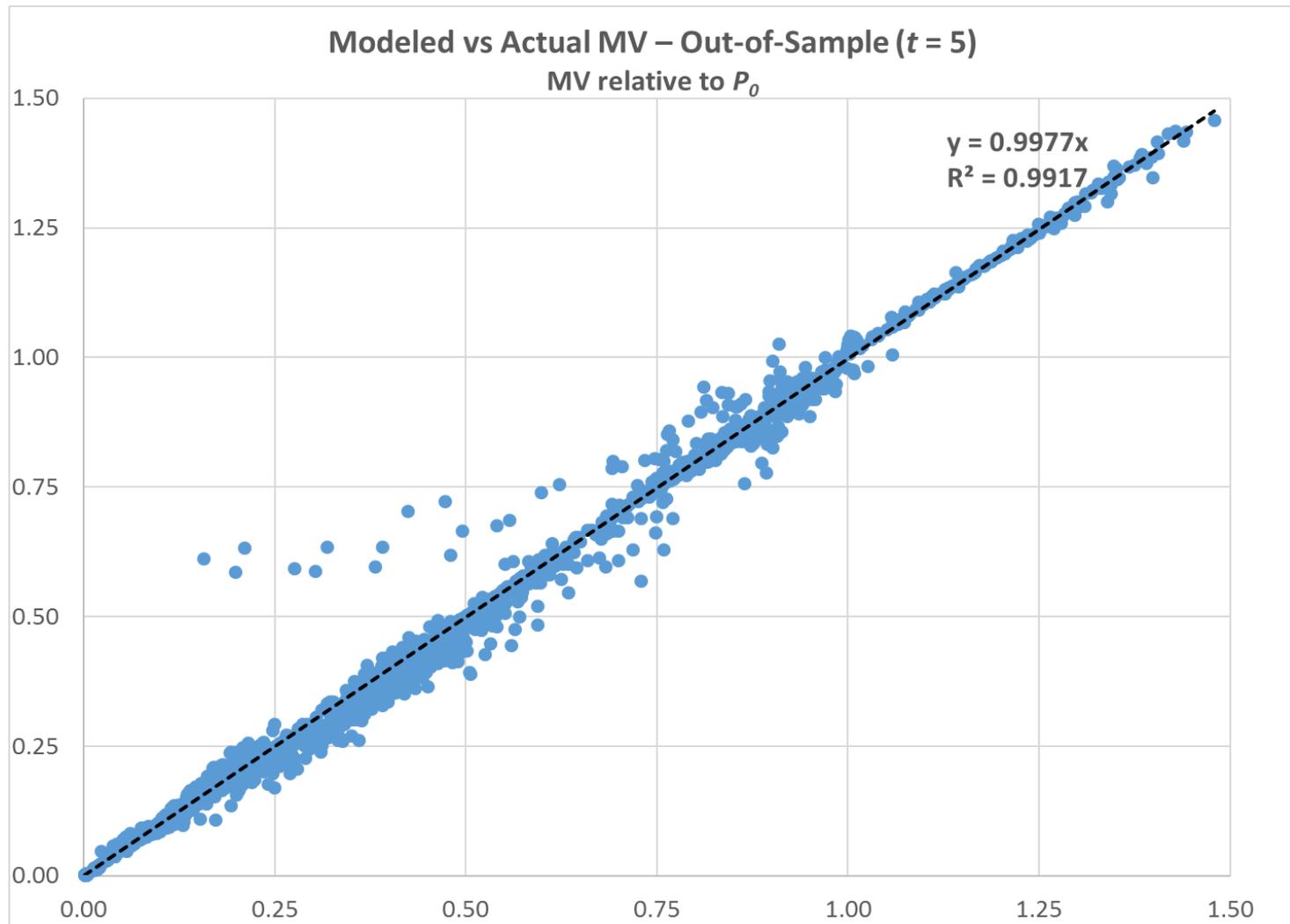
For each CUSIP, compared modeled values of  $MV_t$  at  $t = 2$  and  $t = 5$  to values calculated using nested Monte Carlo simulation

- » Comparisons based on 50 outer loop scenarios at each horizon, i.e. 100 outer loop scenarios in total
  - Outer loop scenarios were generated using a different interest rate model (AXIS G2++ model) than the ESG 2-factor BK model used for GLM calibration
  - Generated 1,000 G2++ scenarios and selected 50 at each horizon ( $t = 2$  and  $t = 5$ ) using stratified sampling to ensure a representative selection.
- » For each outer loop, generated 200 forward-starting risk-neutral inner loop scenarios and calculated the mean of  $PVFC_{i,t}$  across inner loop scenarios
  - Inner loop scenarios were generated using the same ESG 2-factor BK model used for GLM calibration, but starting with yield curves from the outer loop scenarios
  - 880,000 calculations of  $PVFC_{i,t}$  across 44 CUSIPs, 100 outer loops and 200 inner loops
- » For each outer loop, compared modeled values of  $MV_t = g^{-1}(\theta \cdot x_t)$  to actual values (mean of  $PVFC_{i,t}$ )
  - 4,400 comparisons altogether (44 CUSIPs, 100 outer loops)

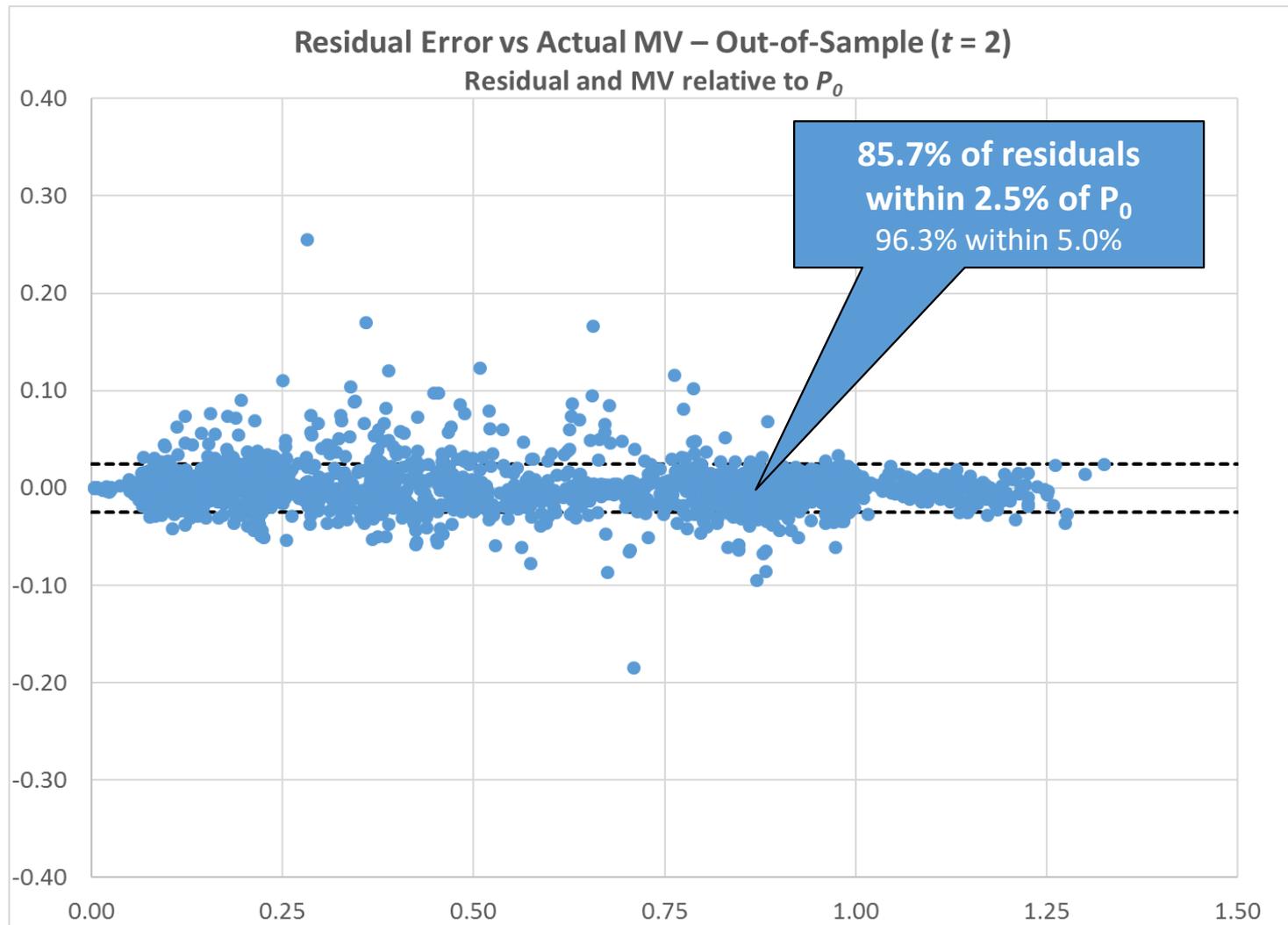
# Out-of-Sample Testing ( $t = 2$ years)



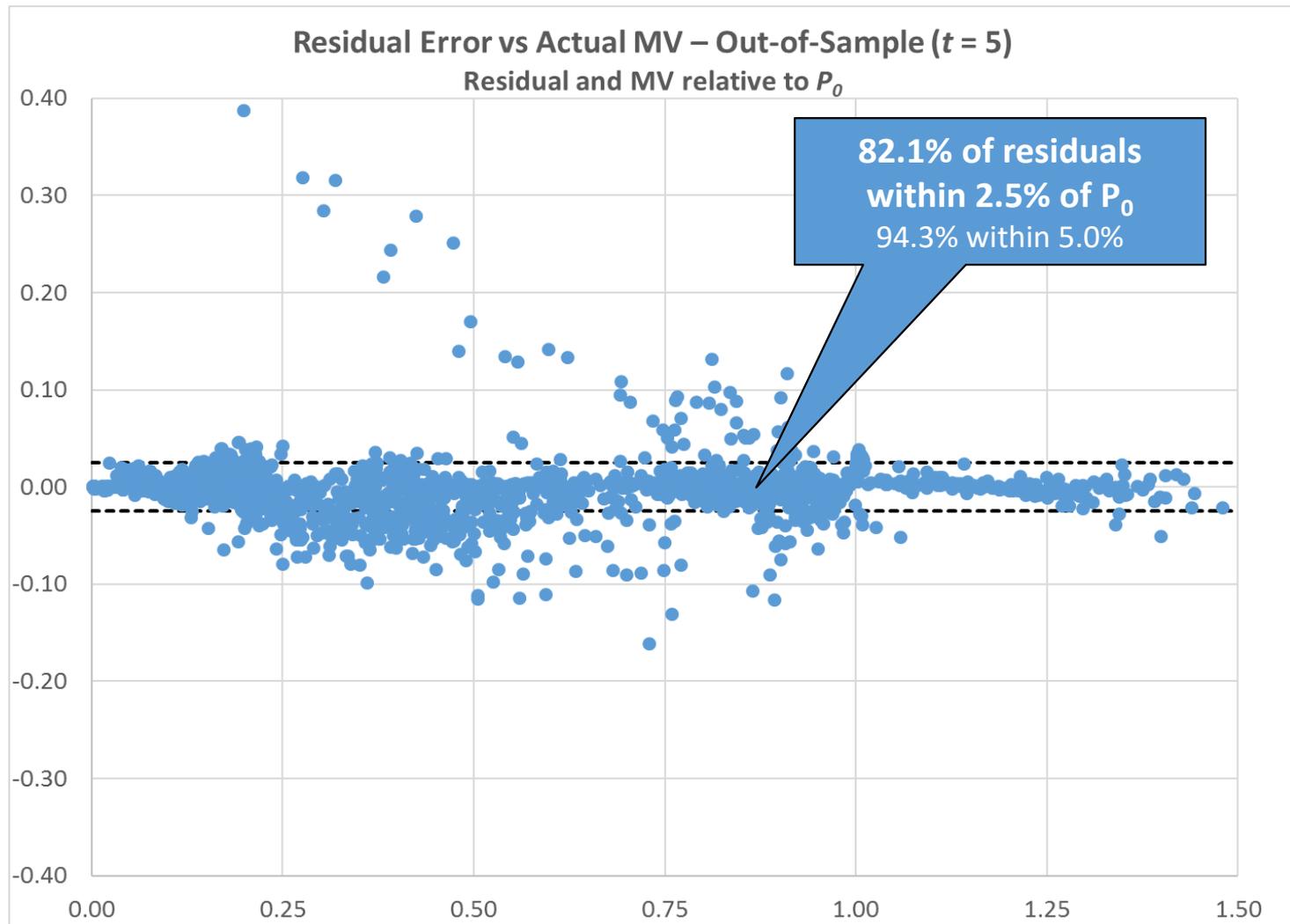
# Out-of-Sample Testing ( $t = 5$ years)



# Out-of-Sample Residuals ( $t = 2$ years)



# Out-of-Sample Residuals ( $t = 5$ years)



# Conclusions and Observations

- » Proposed model is a very good approximation to computationally-intensive forward-starting nested Monte Carlo simulations.
  - Test results may understate the accuracy of the methodology, because the test sample consists of CUSIPs with highly variable cashflows,
- » Almost all the computational effort involves generating the initial set of scenarios and cashflow projections, i.e.  $x_{i,t}$  and  $PVFC_{i,t}$  starting at  $t = 0$ .
  - These values are available as a by-product if Monte Carlo simulation is used to calculate the initial market value  $MV_0$
  - GLM solvers are highly efficient. Once  $x_{i,t}$  and  $PVFC_{i,t}$  have been generated then solving for the parameter vector  $\theta$  takes only a few seconds per CUSIP.
  - Not necessary to recalibrate  $\theta$  with each new production cycle. Can re-use previous calibrations.
  - Calculating out-of-sample estimates of  $MV_t \approx g^{-1}(\theta \cdot x_t)$  is practically instantaneous.



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