Projecting Structured Asset MVs in AXIS

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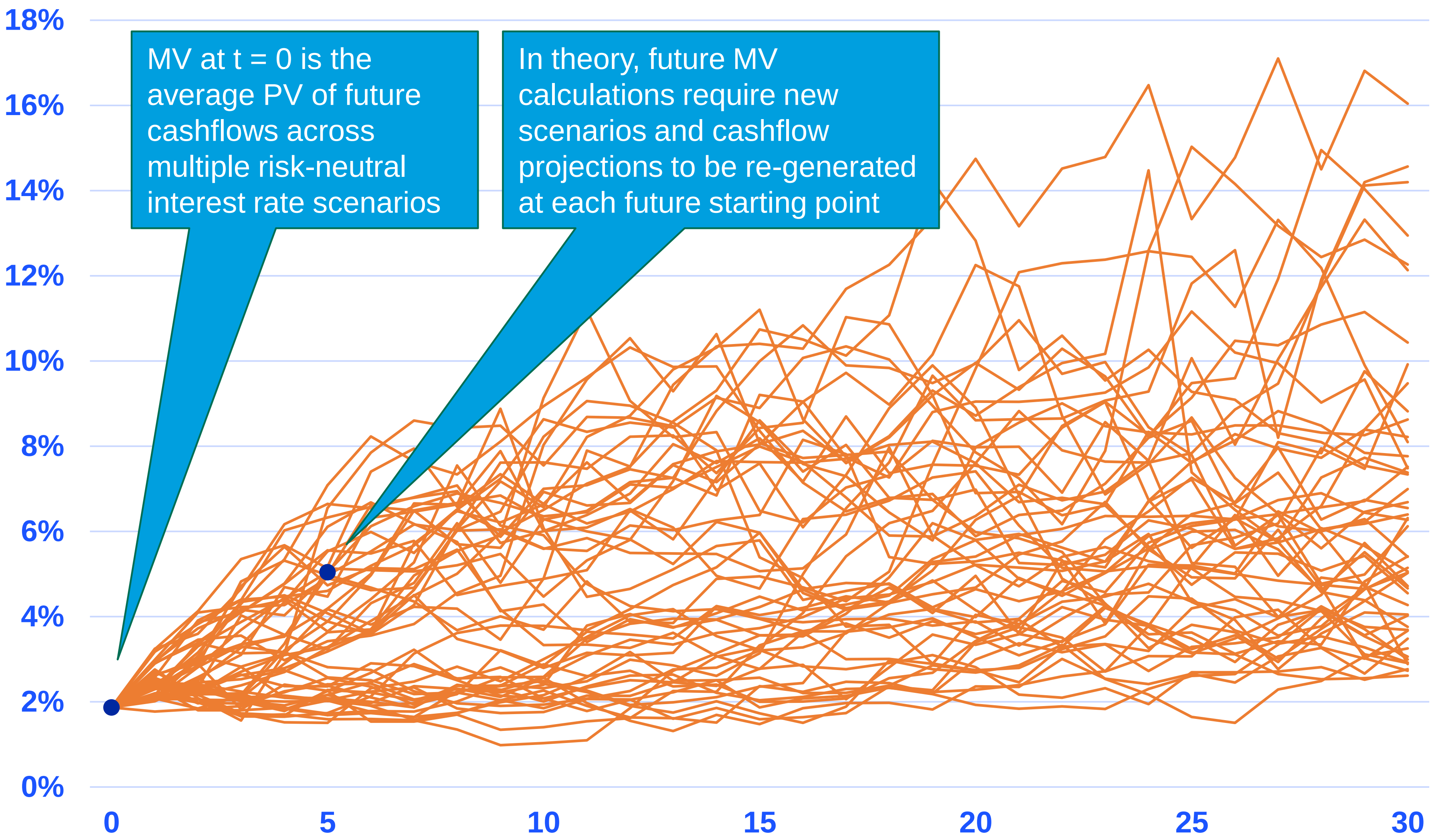
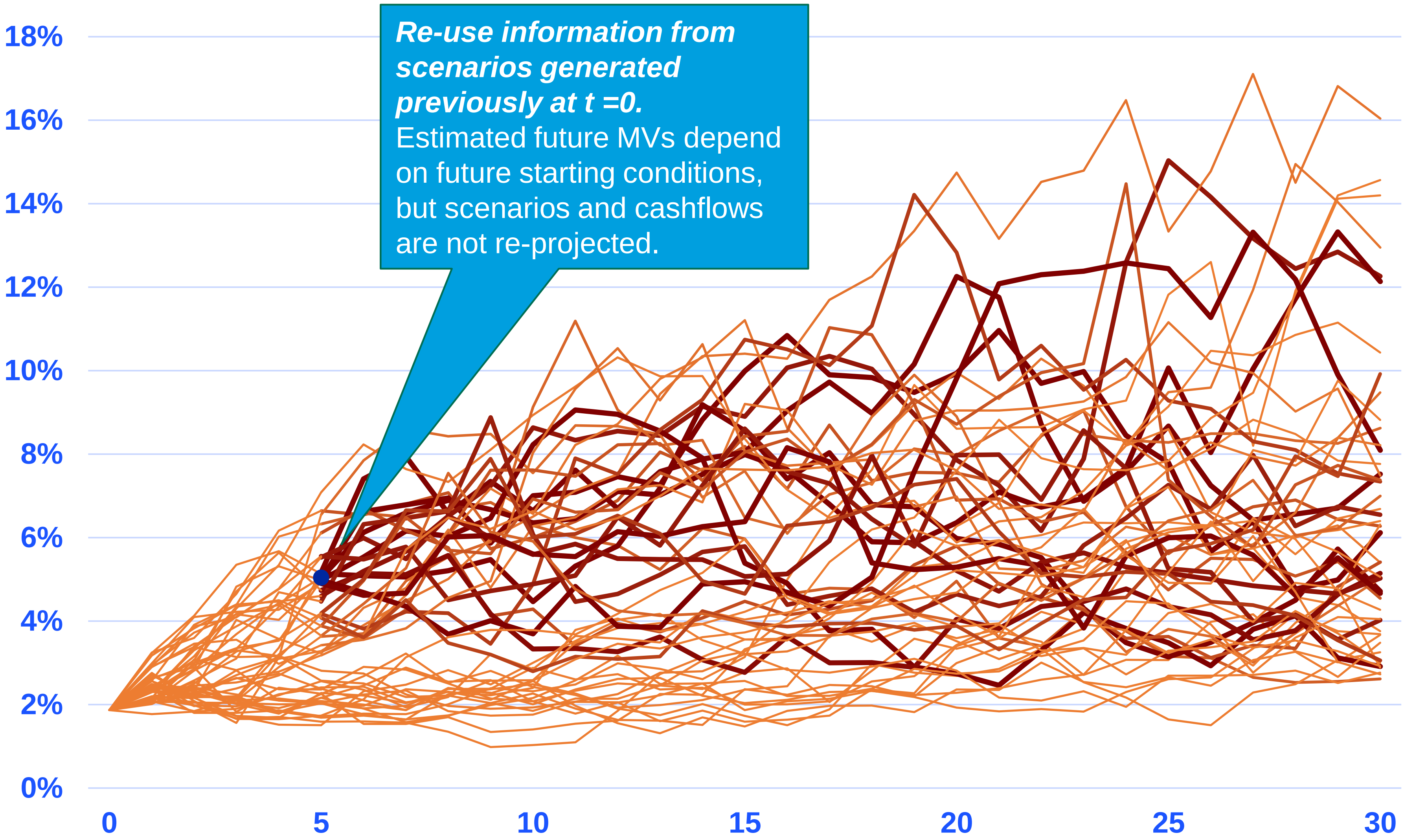
Background

- AXIS is a multi-year projection system for life insurers' assets and liabilities
  - Projections include future cashflows, earnings and balance sheets (BV and MV)
  - Based on user-specified economic scenarios
- AXIS is able to link to MA's Structured Finance API to project asset cashflows, but the SF API is not designed to produce future MV projections
  - SF API can calculate MVs at the projection start date, but not at future dates.
- In theory, accurate MV projections require nested Monte Carlo simulations at each future valuation date. In practice this requires more computational resources than most users have available.
  - Currently, AXIS calculates future MVs by using the SF API to project cashflows along a single implied-forward interest rate path.
- MA has now developed a methodology for approximating nested Monte Carlo MV calculations in AXIS with dramatically improved run-times.
  - Also provides rapid estimates of effective duration and convexity
  - Based on proxy functions, calibrated using SF API cashflow projections
  - Implementation is currently in progress

Illustrative Example: Simulated UST 10y Rates



Alternative to Nested Monte Carlo Simulation



Proxy Functions – Least Squares Monte Carlo

Objective: Estimate MV_t at some future time $t > 0$ as a function of a given vector of state variables $\boldsymbol{x}_t$ (future UST rates, outstanding principal, etc.)

In general, $MV_t = \mathbb{E}^{\mathrm{Q}}(PVFC_t|\boldsymbol{x}_t)$ where $PVFC_t$ is the present value of future cashflows as seen at time t.

Algorithm:

- Estimate MV_0 by generating Monte Carlo simulations of $\boldsymbol{x}_{i,t}$ and $PVFC_{i,t}$ starting at $t = 0$. Then MV_0 is the mean of the simulated values of $PVFC_{i,0}$.
- Assume that $MV_t \approx f(\boldsymbol{x}_t; \boldsymbol{\theta})$ where f is a given parametric function (a proxy function) and $\boldsymbol{\theta}$ is vector of function parameters
- Since $MV_t = \mathbb{E}^{\mathrm{Q}}(PVFC_t|\boldsymbol{x}_t)$, the parameters $\boldsymbol{\theta}$ can be estimated by least squares regression of the simulated values of $PVFC_{i,t}$ against $f(\boldsymbol{x}_{i,t}; \boldsymbol{\theta})$.
  - In effect, the simulated values of $\boldsymbol{x}_{i,t}$ and $PVFC_{i,t}$ provide a very large number of very poor estimates of $\mathbb{E}^{\mathrm{Q}}(PVFC_t|\boldsymbol{x}_t)$.
  - Solve for $\boldsymbol{\theta}$ to minimize $\sum_{i,t}\left(PVFC_{i,t} - f(\boldsymbol{x}_{i,t}; \boldsymbol{\theta})\right)^2$
- Well-established technique (Longstaff & Schwartz, 2001)

Model Details

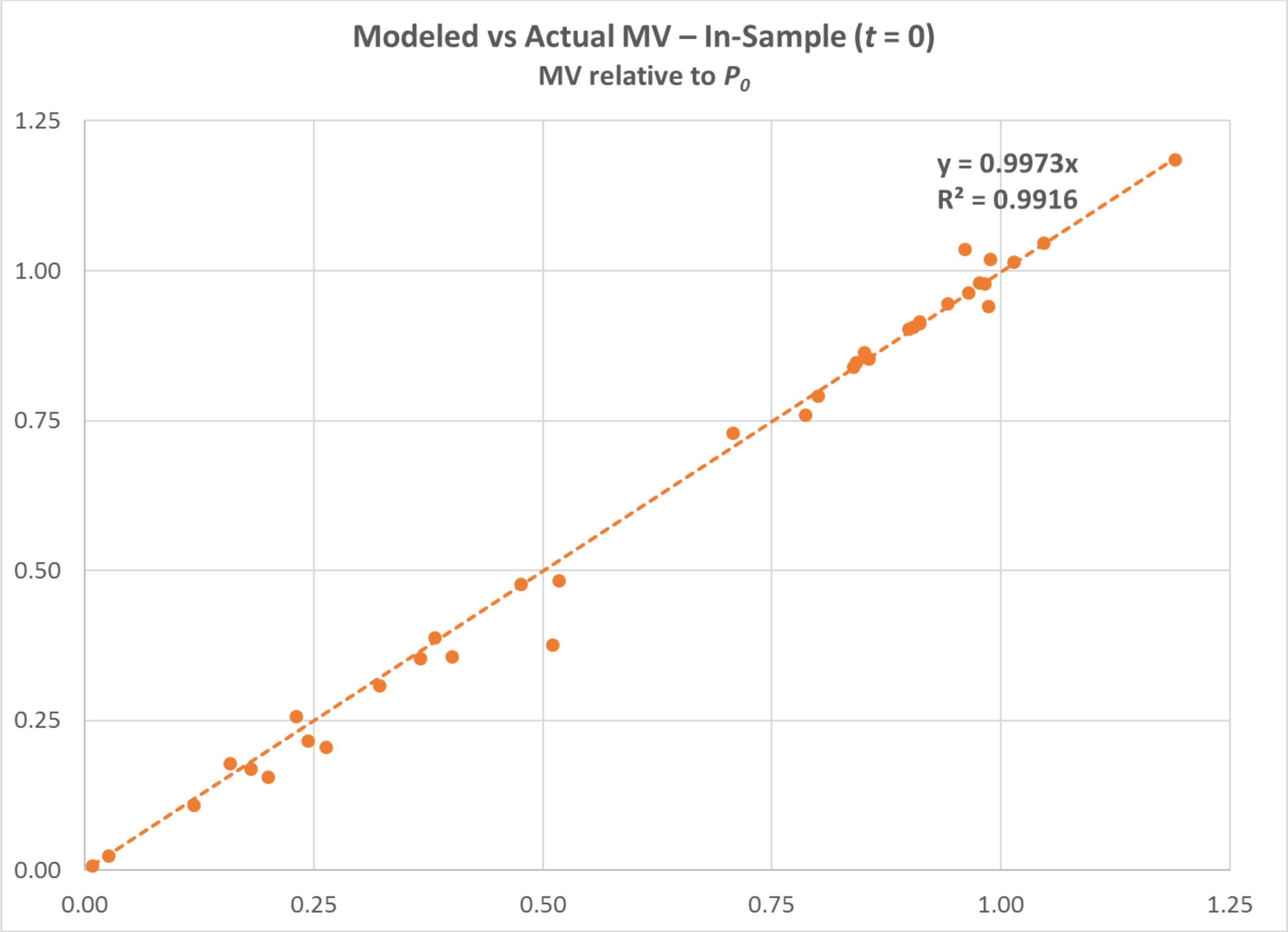
- Assume that $g(MV_t) \approx \boldsymbol{\theta} \cdot \boldsymbol{x}_t$ where $g(MV_t) = \log(\exp(MV_t) - 1)$
  - Implies that $MV_t \approx g^{-1}(\boldsymbol{\theta} \cdot \boldsymbol{x}_t) = \log(\exp(\boldsymbol{\theta} \cdot \boldsymbol{x}_t) + 1)$
  - Ensures that the modeled value of MV_t is always positive even if $\boldsymbol{\theta} \cdot \boldsymbol{x}_t$ is negative, since $g^{-1}(y) \to y$ as $y \to +\infty$ but $g^{-1}(y) \to 0$ as $y \to -\infty$.
- This is a ***generalized linear model***
  - A generalized linear model is *not* a linear model. The link function g is non-linear, and the state vector $\boldsymbol{x}_t$ can include polynomial and interaction terms.
- Predictor variables (state variables):
  - $a_t = P_t/P_0$ where P_t is the outstanding bond principal at time t
  - b_t = 10y US Treasury rate
  - c_t = 1y/10y US Treasury spread
  - d_t = 6m USD LIBOR/Treasury spread
  - $\boldsymbol{x}_t = \{1,\ t,\ t^2, a_t, b_t, c_t, d_t, \ldots\ t^m a_t^n,\ t^m b_t^n,\ t^m c_t^n, t^m d_t^n\}$ where $m, n \leq 2$ (27 terms)
    - $PVFC_{i,t}$ is discounted using Treasury rates plus a spread, but for a given bond the discount spread is constant for all i and t, and is not an explicit predictor variable
- Parameters $\boldsymbol{\theta}$ can be constrained to match $g^{-1}(\boldsymbol{\theta} \cdot \boldsymbol{x}_0)$ to a given value of MV_0

Model Calibration Process

- Tested on 44 ABS and RMBS CUSIPs from actual client portfolios
  - Selected CUSIPs all had highly variable cashflows
- Moody's Analytics ESG 2-factor BK model used to generate 1,000 risk-neutral interest rate paths for US Treasuries and USD LIBOR
- SAV API and Moody's Analytics MPA/PA prepayment and default models used to project cashflows and principal balances along each path
  - Projected at monthly intervals until maturity or until $P_t < 0.05\ P_0$
  - Average of about 50,000 simulated values of $\boldsymbol{x}_{i,t}$ and $PVFC_{i,t}$ for each CUSIP
- Solved for $\boldsymbol{\theta}$ using a standard GLM package
  - Used stepwise GLM regression to identify irrelevant predictors. Not all predictors were relevant to all CUSIPs, but each predictor was relevant to the majority of CUSIPs.
  - Also considered ordinary least squares (OLS) regression as an alternative to GLM regression. OLS is easier to implement, but results were less accurate.
    - OLS solves for $\boldsymbol{\theta}$ to minimize $\sum_{i,t}\left(g(PVFC_{i,t}) - \boldsymbol{\theta} \cdot \boldsymbol{x}_{i,t})\right)^2$ instead of $\sum_{i,t}\left(PVFC_{i,t} - g^{-1}(\boldsymbol{\theta} \cdot \boldsymbol{x}_{i,t})\right)^2$.
- Regression R^2 varied from 20.8% to 93.2%
  - A low R^2 does not necessarily imply a poor quality model. The purpose of the model is to estimate $MV_t = \mathbb{E}^{\mathrm{Q}}(PVFC_t|\boldsymbol{x}_t)$, not $PVFC_{i,t}$. The latter is a random variable that can have many different values for given predictor vector $\boldsymbol{x}_t$.

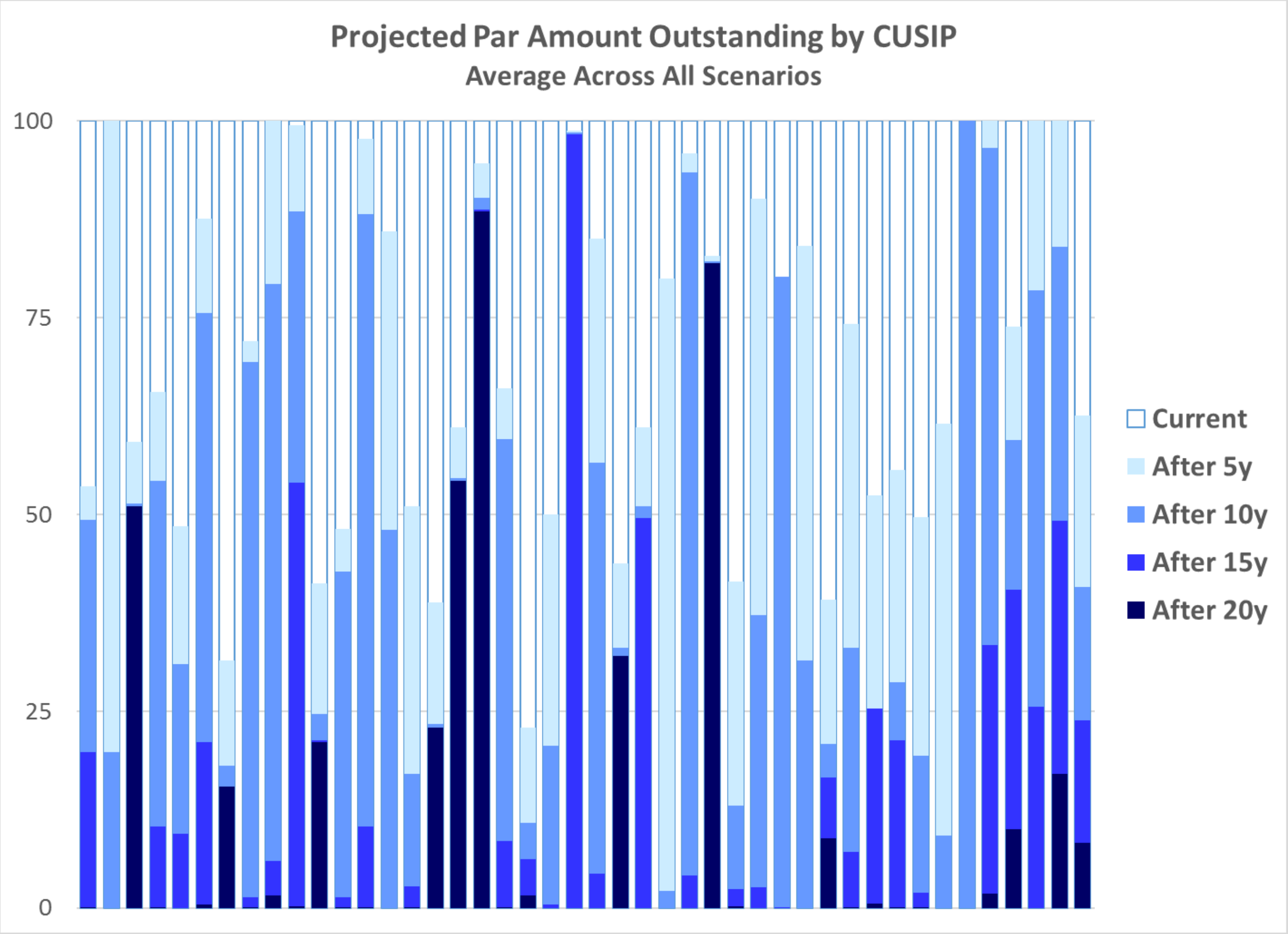
In-Sample Testing

- Compared modeled and actual values of MV_0 (mean of $PVFC_{i,0}$) for each CUSIP



Projected Par Amounts Outstanding

» Most CUSIPs in test sample pay down over 5-15 years

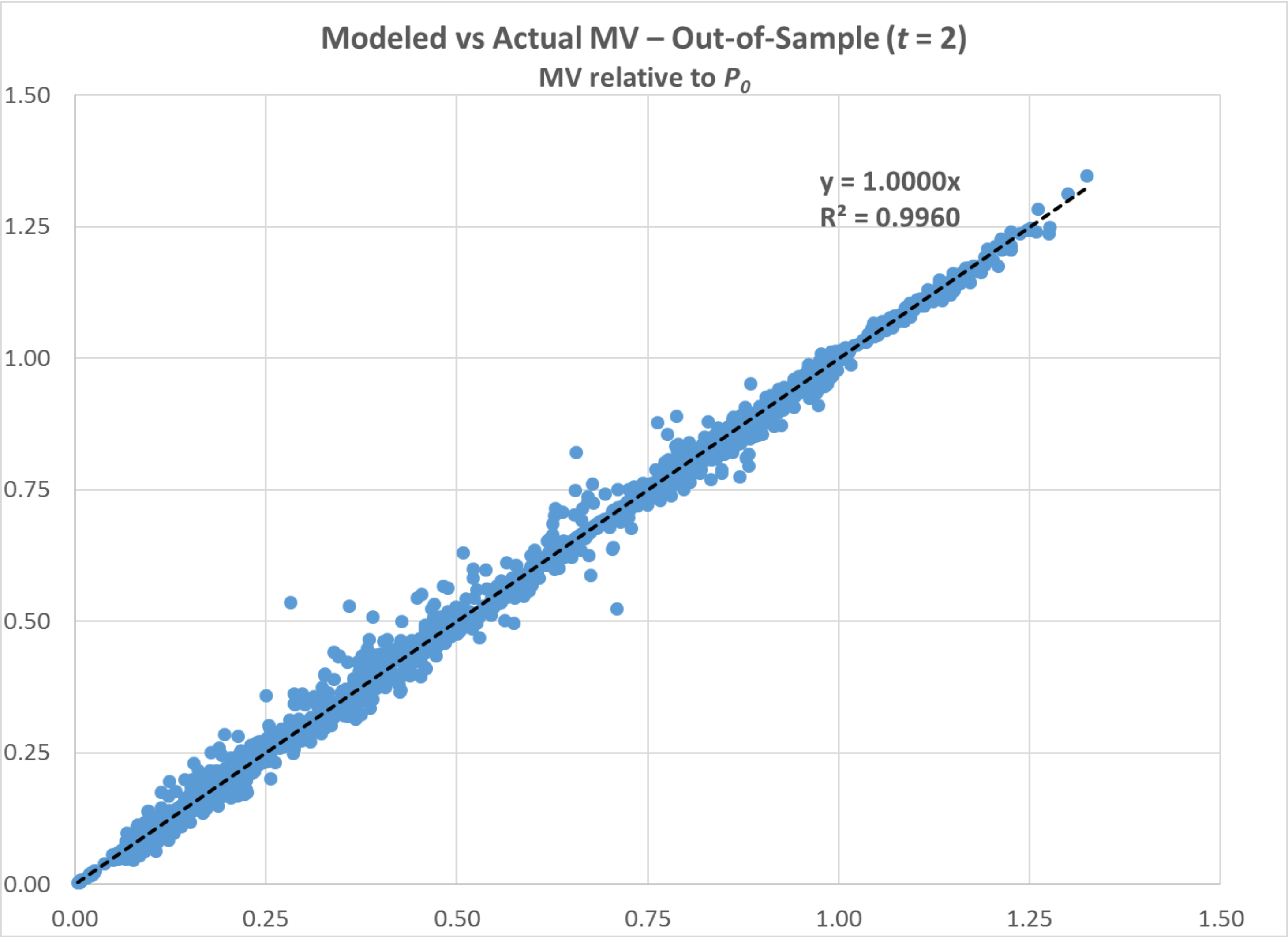


Out-of-Sample Testing

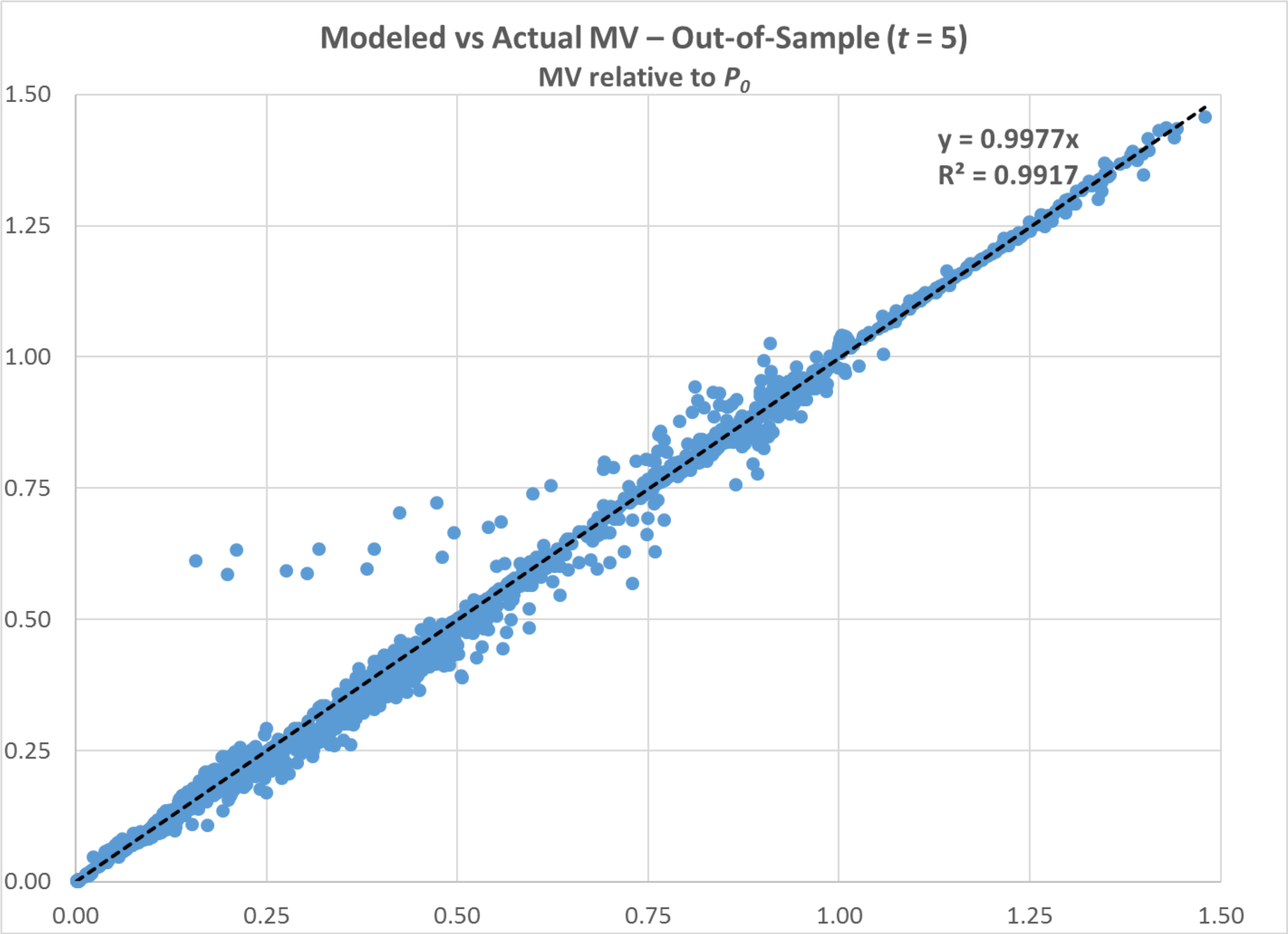
For each CUSIP, compared modeled values of MV_t at $t = 2$ and $t = 5$ to values calculated using nested Monte Carlo simulation

- Comparisons based on 50 outer loop scenarios at each horizon, i.e. 100 outer loop scenarios in total
  - Outer loop scenarios were generated using a different interest rate model (AXIS G2++ model) than the ESG 2-factor BK model used for GLM calibration
  - Generated 1,000 G2++ scenarios and selected 50 at each horizon ($t = 2$ and $t = 5$) using stratified sampling to ensure a representative selection.
- For each outer loop, generated 200 forward-starting risk-neutral inner loop scenarios and calculated the mean of $PVFC_{i,t}$ across inner loop scenarios
  - Inner loop scenarios were generated using the same ESG 2-factor BK model used for GLM calibration, but starting with yield curves from the outer loop scenarios
  - 880,000 calculations of $PVFC_{i,t}$ across 44 CUSIPs, 100 outer loops and 200 inner loops
- For each outer loop, compared modeled values of $MV_t = g^{-1}(\boldsymbol{\theta} \cdot \boldsymbol{x}_t)$ to actual values (mean of $PVFC_{i,t}$)
  - 4,400 comparisons altogether (44 CUSIPs, 100 outer loops)

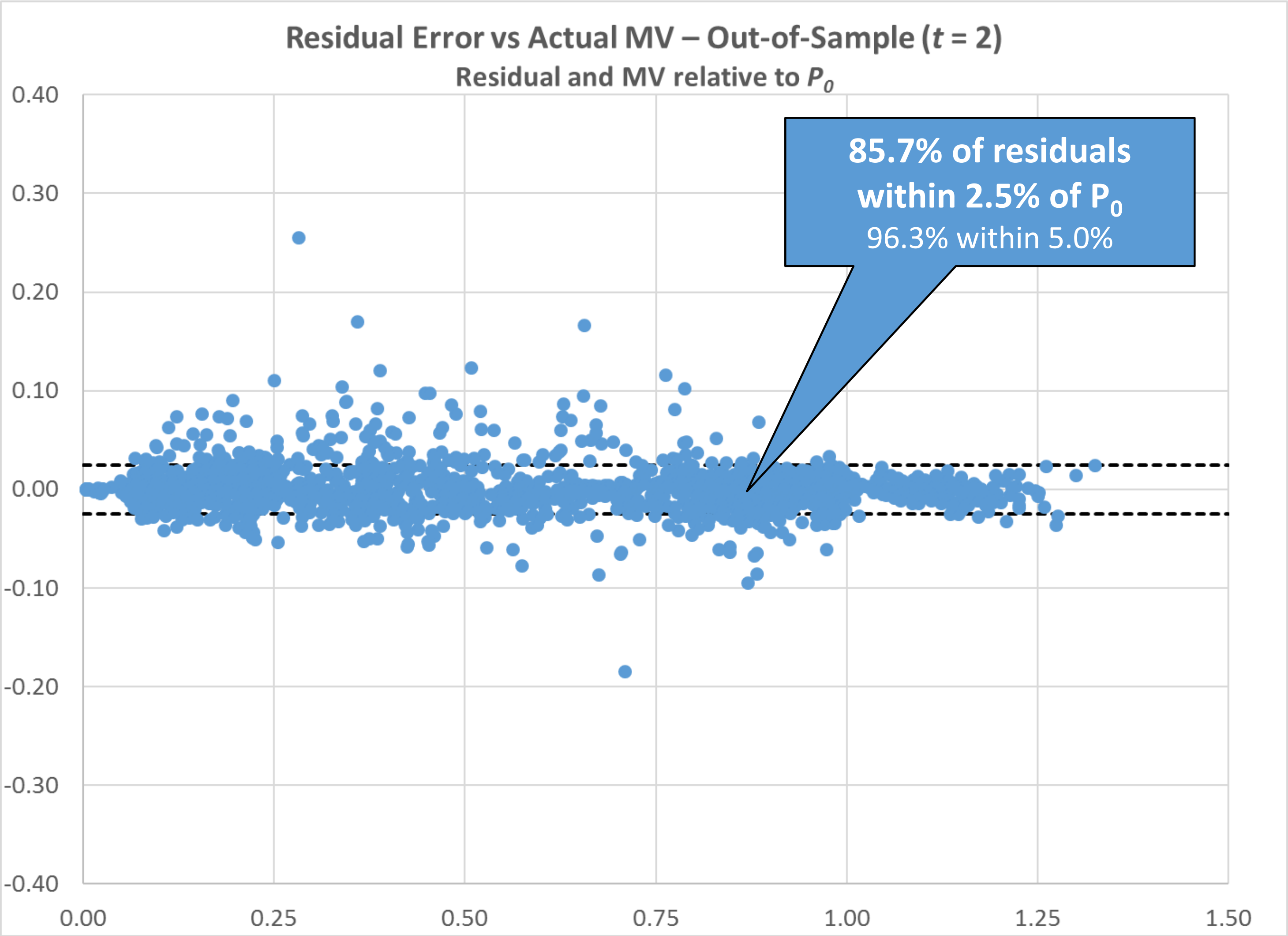
Out-of-Sample Testing ($t = 2$ years)



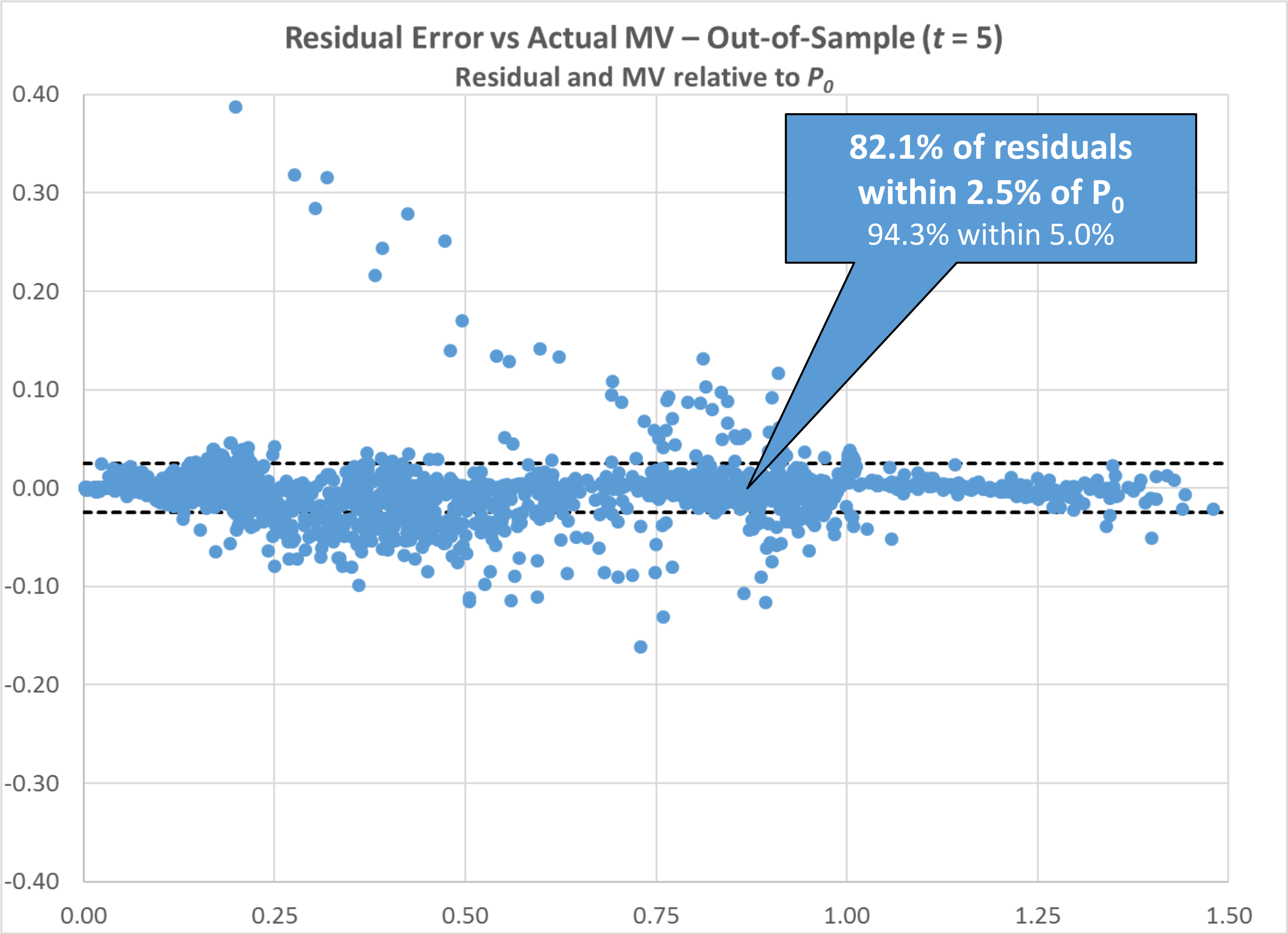
Out-of-Sample Testing ($t = 5$ years)



Out-of-Sample Residuals ($t = 2$ years)



Out-of-Sample Residuals (t = 5 years)



Conclusions and Observations

- Proposed model is a very good approximation to computationally-intensive forward-starting nested Monte Carlo simulations.
  - Test results may understate the accuracy of the methodology, because the test sample consists of CUSIPs with highly variable cashflows,
- Almost all the computational effort involves generating the initial set of scenarios and cashflow projections, i.e. $\boldsymbol{x}_{i,t}$ and $PVFC_{i,t}$ starting at $t = 0$.
  - These values are available as a by-product if Monte Carlo simulation is used to calculate the initial market value MV_0
  - GLM solvers are highly efficient. Once $\boldsymbol{x}_{i,t}$ and $PVFC_{i,t}$ have been generated then solving for the parameter vector $\boldsymbol{\theta}$ takes only a few seconds per CUSIP.
  - Not necessary to recalibrate $\boldsymbol{\theta}$ with each new production cycle. Can re-use previous calibrations.
  - Calculating out-of-sample estimates of $MV_t \approx g^{-1}(\boldsymbol{\theta} \cdot \boldsymbol{x}_t)$ is practically instantaneous.

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